FUZZY PROGRAMMING APPROACH FOR A COMPROMISE ALLOCATION OF REPAIRABLE COMPONENTS

Irfan Ali* and S. Suhaib Hasan

ABSTRACT
The present paper, considered the allocation problem of repairable components for a parallel-series system as a multi-objective optimization problem for two different models. In the first model, the reliability of subsystems are considered as different objectives. While in the second model, the cost and time spent on repairing the components are considered as two different objectives. Selective maintenance operation is used to select the repairable components and a fuzzy programming algorithm is used to obtain compromise allocation of repairable components for the two models under some given constraints. A numerical example is also given to illustrate the procedure.

Key Words: Reliability, Fuzzy Programming, Compromise allocation, Selective Maintenance, Multi-objective programming.

.

----------------------*---------------------

1. INTRODUCTION

In every industry, systems are used in the production of goods. If such systems deteriorate or fail, then effect can be wide spread. System deterioration is often reflected in higher production cost, time lower product quality and quantity. The system maintenance decision is taken on the basis of the state condition of the system (*i.e.* whether the system is good or bad). The aim is to present a model of reliability improvement maintenance policies that minimizes the total cost and time spent on maintaining a system. For this purpose, we consider a system which is a series arrangement of m subsystems and performing a sequence of identical production runs.

Suppose that after completion of a particular production run, each component in the system is either functioning or failed. Ideally all the failed components in the subsystems are repaired and then replaced back prior to the beginning of the next production run. However, due to constraints on time and cost, it may not be possible to repair all the failed components in the system. In such situation, a method is needed to decide which failed components should be repaired and replaced back prior to the next production run and the rest be left in a failed condition. This process is referred to as selective maintenance (See Rice et al. 1998). In the selective maintenance the number of components available for the next production run in the i^{th} subsystem will be

$$(n_i - a_i) + d_i\ ,\ i = 1, 2, ..., m \tag{1}$$

The reliability of the given system is defined as

$$R = \prod_{i=1}^{m} \left\{ 1 - (1 - r_i)^{n_i - a_i + d_i} \right\} \tag{2}$$

The repair time constraint for the system is given as

$$\sum_{i=1}^{m} t_i \left[d_i + \exp(\theta_i d_i) \right] \le T_0 \tag{3}$$

where t_i is the time required to repair a component in $i-th$ subsystem and $\exp(\theta_i\, a_i)$ is the additional time spent due to the interconnection between parallel components (Wang *et al.* (2009)).

The repair cost constraint for the system is defined as

$$\sum_{i=1}^{m} c_i \left[d_i + \exp(\beta_i d_i) \right] \le C_0 \tag{4}$$

where $\exp(\beta_i\, a_i)$ is the additional cost spent due to the interconnection between parallel components (Wang *et al.* (2009)).

However, in the event when the reliability of each subsystems are of equally serious concern. Let us consider, for instance, the following multi-objective problem (see Ali et al. (2011c)):

$$\left.\begin{array}{ll} \text{Maximize } (R_1, R_2, \ldots, R_m) & (i) \\ \text{subject to} & \\ \sum_{i=1}^{m} t_i \left[d_i + \exp(\theta_i d_i) \right] \le T_0 & (ii) \\ \sum_{i=1}^{m} c_i \left[d_i + \exp(\beta_i\, d_i) \right] \le C_0 & (iii) \\ 0 \le d_i \le a_i\ , i = 1, 2, ..., m \text{ and integers} & (iv) \end{array}\right\} \tag{5}$$

where $R_i = \left\{1-(1-r_i)^{n_i - a_i + d_i}\right\}, \; i = 1, 2, ..., m.$

Ali et al. (2011c) also discussed the situation in which time taken and the cost spent on system maintenance are minimized simultaneously for the required reliability R^* (say). The mathematical model of the problem is defined as:

$$\left.\begin{aligned} &Min\, C = \sum_{i=1}^{m} c_i \left[d_i + \exp\left(\beta_i d_i\right)\right] \\ &\text{and} \\ &Min\, T = \sum_{i=1}^{m} t_i \left[d_i + \exp\left(\theta_i d_i\right)\right] \\ &\text{subject to} \\ &\prod_{i=1}^{m} 1-\left(1-r_i\right)^{n_i - a_i + d_i} \geq R^* \\ &0 \leq d_i \leq a_i,\, i = 1,2,...,\, m \text{ and integers} \end{aligned}\right\} \quad (6)$$

The selective maintenance operation is an optimal decision-making activity for systems consisting of several components under limited maintenance duration. The main objective of the selective maintenance operation is to select the most important component in subsystem. Rice et al. (1998) were the first to deal with the selective maintenance problem. Ali *et al.* (2011a, 2011b, 2011c) and Faisal and Ali (2012) considered the problem of optimum allocation of repairable and replaceable components for series-parallel system by using selective maintenance.

Fuzzy programming offers a powerful means of handling optimization problems with fuzzy parameters. Fuzzy programming has been used in different ways in the past. In (2010), Mahapatra *et al.* studied the fuzzy programming approach to stochastic transportation problem and many others. In reliability Park (1987), Mahapatra and Roy (2006), Huang (1997), Dhingra (1992), Rao and Dhingra (1992) and Ravi et al. (2000) have used fuzzy multi-objective optimization method to solve reliability optimization problem having several conflicting objectives.

This paper presents a new contribution in the field of system reliability optimization for compromise allocation problem of repairable components. A Fuzzy programming algorithm is used to obtain compromise allocation of repairable components.

The multi-objective NLPPS are solved by "Fuzzy programming algorithm" using software package LINGO. LINGO is a user's friendly package for constrained optimization developed by LINDO Systems Inc. A user's guide- LINGO User's Guide (2001) is also available. For more information one can visit the site http://www.lindo.com.

2. THE FUZZY PROGRAMMING APPROACH

To solve multi-objective allocation problem of repairable components defined in equation (5), we apply the fuzzy programming approach (see Mahapatra et al. (2010)). At first, we find the upper bound U_r (best) and lower bound L_r (worst) for corresponding objective function Z_r, $r = 1, 2, \ldots, m$.

Let U_r = aspiration level of achievement for objective r,

L_r = lowest acceptable level of achievement for objective r,

$\delta_r = U_r - L_r$ = the degradation allowance for objective r,

when the aspiration level and degradation allowance for each objective are specified.

Algorithm:

Step 1: Solve multi-objective allocation problem of repairable components as a single objective used each time and all other ignored.

Step 2: Determine the corresponding values for every objective at each solution derived.

Step 3: Construct a pay-off matrix, according to every objective w. r. to each solution the pay-off matrix in the main program gives the set of non dominated solution which shown in the following table.

$$\begin{bmatrix} & Z_1 & Z_2 & \ldots & Z_m \\ & & & & \\ d^{(1)} & Z_{11} & Z_{12} & \ldots & Z_{1m} \\ d^{(2)} & Z_{21} & Z_{22} & \ldots & Z_{2m} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ d^{(m)} & Z_{m1} & Z_{m2} & \ldots & Z_{mm} \end{bmatrix}$$

where $d^{(1)}, d^{(2)}, \ldots, d^{(m)}$ is the ideal solution for the objective $Z_1, Z_2 \ldots Z_m$ respectively.

Let $Z_{ij} = Z_i(d^j), i = 1, 2, \ldots, m$ *and* $j = 1, 2, \ldots, m$ are the maximum value (best) for each objective Z_i, $i = 1, 2, \ldots, m$.

Step 4: To find the best (U_r) and worst (L_r) for each objectives corresponding to the set of solution i.e. $U_r = Z_{rr}$ *and* $L_i = \underset{r\geq 1}{Min}\{Z_{1r}, Z_{2r}, \ldots, Z_{mr}\}$. To satisfy, $Z_r \geq U_r$, $r = 1, 2, \ldots, m$ and constraints (*ii*), (*iii*) and (*iv*) of NLPP (5).

Step 5: Construct membership function as,

$$\Omega_{Z_r}(d_{ij}) = \begin{cases} 0 & Z_r < L_r \\ \dfrac{Z_r - L_r}{U_r - L_r} & L_r < Z_r < U_r \\ 1 & Z_r \geq U_r \end{cases}$$

If $\Omega_{Z_r}(d_{ij}) = 1$; then Z_r is perfectly achieved,

$=0$; Z_r is nothing achieved,

$0 \leq \Omega_{Z_r}(d_{ij}) \leq 1$; then Z_r is partially achieved.

Step 6: Let $\delta_r = \dfrac{Z_r - L_r}{U_r - L_r}, r = 1, 2, \ldots, m$

Using max-min/min-max operator, we have

$\max[\min(\delta_1, \delta_2, \ldots, \delta_m)]$,

then we have; $Max\ \delta$

$$\delta_1 \geq \delta$$
$$\delta_2 \geq \delta$$
$$\vdots$$
$$\delta_m \geq \delta$$

$$where\ \delta = \underset{r}{Max}\begin{Bmatrix} \Omega_{Z_r}(d_{ij}); \\ i = j = 1, 2, \ldots, m \,\&\, r = 1, 2, \ldots, m \end{Bmatrix}$$

Finally we obtained the mathematical programming formulation for equation (5) through fuzzy programming , as follows:

$$\left.\begin{array}{l} Maximize \quad \delta \\ \\ \text{subject to} \\ \left\{1-(1-r_i)^{n_i-a_i+d_i}\right\} - \delta(U_i - L_i) \geq L_i \\ \sum_{i=1}^{m} t_i\left[d_i + \exp(\theta_i d_i)\right] \leq T_0 \\ \sum_{i=1}^{m} c_i\left[d_i + \exp(\beta_i\, d_i)\right] \leq C_0 \\ \qquad \delta \geq 0 \\ 0 \leq d_i \leq a_i\,, i = 1,2,\ldots,\, m \text{ and integers} \end{array}\right\} \qquad (7)$$

In a similar way, we solve multi-objective allocation problem of repairable components defined in equation (6); we apply the fuzzy programming approach. At first, we find the upper bound L_r (best) and lower bound U_r (worst) for corresponding objective function Z_r where $r = 1, 2, \ldots, k$.

Let L_r = aspiration level of achievement for objective r ,

U_r = highest acceptable level of achievement for objective r ,

$\delta_r = U_r - L_r$ = the degradation allowance for objective r ,

when the aspiration level and degradation allowance for each objective are specified.

Now construct membership function for model (2) defined in equation (6) as,

$$\Omega_{Z_r}(d_{ij}) = \begin{cases} 1 & Z_r < L_r \\ \dfrac{U_r - Z_r}{U_r - L_r} & L_r < Z_r < U_r \\ 0 & Z_r \geq U_r \end{cases}$$

If $\Omega_{Z_r}(d_{ij}) = 1$; then Z_r is perfectly achieved,

$=0$; Z_r is nothing achieved,

$0 \leq \Omega_{Zr}(d_{ij}) \leq 1$; then Z_r is partially achieved.

Let us define $\delta_r = \dfrac{U_r - Z_r}{U_r - L_r}, r = 1, 2, \ldots, k$

Next using max-min/min-max operator, we have

$\max[\min(\delta_1, \delta_2, \ldots, \delta_k)]$,

then we have; $Max\ \delta$

$$\delta_1 \geq \delta$$
$$\delta_2 \geq \delta$$
$$\vdots$$
$$\delta_k \geq \delta$$

where

$$\delta = \underset{r}{Max}\left\{\Omega_{Z_r}(d_{ij}); i = j = 1, 2, \ldots, m \,\&\, r = 1, 2, \ldots, k\right\}$$

Finally we obtained the mathematical programming formulation for equation (6) through fuzzy programming as follows:

$$\left.\begin{array}{l} Maximize\ \delta \\ \text{subject to} \\ \sum_{i=1}^{m} t_i\left[d_i+\exp(\theta_i d_i)\right]+\delta(U_1-L_1)\le U_1 \\ \sum_{i=1}^{m} c_i\left[d_i+\exp(\beta_i\, d_i)\right]+\delta(U_2-L_2)\le U_2 \\ \prod_{i=1}^{m} 1-(1-r_i)^{n_i-a_i+d_i}\ge R^* \\ \delta\ge 0 \\ 0\le d_i\le a_i\,, i=1,2,...,\ m \text{ and integers} \end{array}\right\} \quad (8)$$

3. NUMERICAL ILLUSTRATION

Consider a system consisting of 3 subsystems. The available time between two production runs for repairing and replacing back the components is 60 time units. Let the given maintenance cost of the system be 90 units. The other parameters for the various subsystems are given in table 1.

Table 1: The parameters for the numerical example

Subsystem	1	2	3
n_i	10	8	12
r_i	0.55	0.45	0.50
a_i	7	5	8
c_i	8	7	8
t_i	3	4	3
θ_i	0.25	0.25	0.25
β_i	0.25	0.25	0.25

Solution by Using Fuzzy Programming Approach

Model 1: Using the values given in Table 1 the NLPP (5) and their optimal solutions $\underline{d}^{(i)}$; $i=1,\ 2\ and\ 3$ with the corresponding values of R_i^* are listed below. These values are obtained by software LINGO.

$$\left.\begin{array}{l} Maximize\ R_i=\left\{1-(1-r_i)^{n_i-a_i+d_i}\right\} \\ \qquad i=1,2\dots,m \\ \text{subject to} \\ \sum_{i=1}^{5} t_i\left[d_i+\exp(\theta_i d_i)\right]\le 60 \\ \sum_{i=1}^{5} c_i\left[d_i+\exp(\beta_i\, d_i)\right]\le 90 \\ 1\le d_i\le a_i\,, i=1,2,...,\ 5 \text{ and integers} \end{array}\right\} \quad (9)$$

Using the equation (9) construct a pay-off matrix, according to every objective with respect to each solution the pay-off matrix in the main program gives the set of non dominated solution which shown in the following table

	Z_1	Z_2	Z_3
$d^{(1)}$	0.9992433	0.9084938	0.9687500
$d^{(2)}$	0.9589938	0.9916266	0.9687500
$d^{(3)}$	0.9589938	0.9084938	0.9990234

The upper bounds for the given model 1 are

$U_1=0.9992433, U_2=0.9916266, U_3=0.9990234$ and lower bound

$L_1=0.9589938,\ L_2=9084938, L_3=0.9687500$.

Using the equation (7), we can formulated the model 1as

$$\left.\begin{array}{l} Maximize\quad \delta \\ \\ Subject\ \ to \\ \left\{1-(1-0.55)^{(3+d_1)}\right\}-0.0402495\delta\ge 0.95899 \\ \left\{1-(1-0.45)^{(3+d_2)}\right\}-0.0831328\delta\ge 0.90849 \\ \left\{1-(1-0.55)^{(3+d_i)}\right\}-0.0302734\delta\ge 0.96875 \\ \sum_{i=1}^{m} t_i\left[d_i+\exp(\theta_i d_i)\right]\le 60 \\ \sum_{i=1}^{m} c_i\left[d_i+\exp(\beta_i\, d_i)\right]\le 130 \\ \delta\ge 0 \\ 1\le d_i\le a_i\,, i=1,2,...,\ m\ and\ integers \end{array}\right\} \quad (10)$$

The above problem (10) is solved by the LINGO Software for obtaining the optimal solution of the problem. We get $\delta = 0.7741945$ and the compromise solution as $d_1^* = 3,\ d_2^* = 4,\ d_3^* = 3$. The optimal reliabilities of each subsystem are

$R_1^* = 0.9916962,\ R_2^* = 0.9847756,\ R_3^* = 0.9921875$.

Model 2: Using the values given in Table 1 the NLPP (6) for the desired reliability requirement $R^* \geq 0.97$ has been solved and construct a pay-off matrix, according to every objective with respect to each solution the pay-off matrix in the main program gives the set of non dominated solution which shown in the following table for

$$\begin{bmatrix} & Z_1 & Z_2 \\ d^{(1)} & 62.38 & 141.72 \\ d^{(2)} & 64.66 & 141.30 \end{bmatrix}$$

The upper bound and lower bond for the model 2 are

$$U_1 = 64.66,\ U_2 = 141.72$$

and $L_1 = 62.38,\ L_2 = 141.30$.

$$\left.\begin{aligned} & \textit{Maximize } \delta \\ & \text{subject to} \\ & \sum_{i=1}^{m} t_i\left[d_i + \exp(\theta_i d_i)\right] + 2.38\delta \leq 64.66 \\ & \sum_{i=1}^{m} c_i\left[d_i + \exp(\beta_i d_i)\right] + 0.42\delta \leq 141.30 \\ & \prod_{i=1}^{m} 1 - (1 - r_i)^{n_i - a_i + d_i} \geq 0.97 \\ & \delta \geq 0 \\ & 1 \leq d_i \leq a_i,\ i = 1,2,\ldots,m \text{ and integers} \end{aligned}\right\} \quad (11)$$

The above problem (11) is solved by the LINGO Software for obtaining the optimal solution of the problem. We get $\delta = 0.0233$ and the compromise solution as $d_1^* = 3,\ d_2^* = 4,\ d_3^* = 4$.

6. CONCLUSION

This paper is an attempt to utilize Fuzzy programming approach to the solution of optimum compromise allocation of repairable components in a system. Further, a numerical example is presented to demonstrate the practical utility of the fuzzy programming approach in system reliability.

REFERENCES

[1]. Ali, I., Raghav, Y. S., and Bari, A., (2011); Allocating Repairable and Replaceable Components for a System availability using Selective Maintenance: An Integer Solution; *Safety and Reliability society*, **31(2),** 9-18.

[2]. Ali, I., Khan, M. F., Raghav, Y.S., and Bari, A., (2011). Allocating Repairable and Replaceable Components for a System Availability using Selective Maintenance with probabilistic constraints; *American Journal of Operations Research*, **1(3):**147-154.

[3]. Ali, I., Raghav, Y.S., and Bari, A., (2011c); Integer goal programming approach for finding a compromise allocation of repairable components, *International Journal of Engineering Science and Technology*, **3(6):** 184-195.

[4]. Dhingra, A.K. (1992); Optimal apportionment of reliability and redundancy in series systems under multiple objectives, *IEEE Trans. Rel.*, **41,** 576-582.

[5]. Huang, H.Z. (1997); Fuzzy multi-objective optimization decision-making of reliability of series system, *Microelectronics Reliability*, **37 (3),** 447–449.

[6]. Khan, M. F. and Ali, I. (2012); Allocation Problem of Repairable and Replaceable Components for a System Reliability using Selective Maintenance: An Integer Solution, *International Journal of Scientific and Engineering Research*, **3(5)**, 1-4.

[7]. *LINGO User's Guide* (2001), published by Lindo Systems Inc., 1415 North Dayton Street, Chicago, Illinois-60622 (USA).

[8]. Mahapatra, D. R., Roy, S. K. and Biswal, M. P. (2010); Stochastic Based on Multi-Objective Transportation Problems Involving Normal Randomness, *Advance Modeling and Optimization*, **12 (2),** 205-223.

[9]. Mahapatra, G. S. and Roy, T. K. (2006); Fuzzy Multi-Objective Mathematical Programming on Reliability Optimization Model, *Applied Mathematics and Computation,* **174 (1),** 643-659.

[10]. Park, K. S. (1987); Fuzzy apportionment of system reliability, *IEEE Transactions on Reliability*, **R-36,** 129-132.

[11]. Rice, W.F., Cassady, C.R., and Nachlas, J.A., (1998); Optimal Maintenance Plans under Limited Maintenance Time, Industrial Engineering Research **98** Conference proceedings.

[12]. Rao, S. S. and Dhingra, A. K. (1992); Reliability and redundancy apportionment using crisp and fuzzy multi-objective Optimization approaches, *Reliability Eng. Syst. Safety*, **37**, 253-261.

ISSN 2229-5518

[13]. Ravi, V., Reddy, P. J. and Hans-Jurgen Zimmermann (2000); Fuzzy global optimization of complex system reliability, *IEEE Trans. Rel.,* **8,** 241-248.

[14]. Wang, Z., Chen, T., Tang, K. and Yao, X. (2009); A Multi-objective Approach to Redundancy Allocation Problem in Parallel- series System, *IEEE Congress Evolutionary Computation*, 582-589.

BIOGRAPHICAL NOTES

- **Irfan Ali** received M.Sc. and M. Phil. in Statistics from Aligarh Muslim University, Aligarh, India. Presently he is pursuing Ph.D. from department of Statistics and Operations research Aligarh Muslim University, Aligarh, India. He has published 16 research papers in national and international journals of reputes. He has also presented 4 research articles in national conferences.
- **Suhaib Hasan** received M.Sc. and M. Phil. in Statistics from Aligarh Muslim University, Aligarh, India. He is a Associate Professor in the Department of Statistics and Operations research Aligarh Muslim University, Aligarh, India. Mr. Hasan has more than 25 years of experience in teaching and research. His area of interest also includes linear models and econometrics.